

Fourierova transformacija

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

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Zadrževalnik ničtega reda

$$G_0(s) = \frac{1 - e^{-sT}}{s}$$

Z-transformacija

$$\mathcal{Z}\{x(k)\} = X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$$

$$\mathcal{Z}^{-1}\{X(z)\} = x(k) = \frac{1}{2\pi j} \oint_C X(z) z^{k-1} dz = \sum \text{Res}[X(z) z^{k-1}]; \quad \text{Res}_{z=a} f(z) = \frac{1}{(q-1)!} \lim_{z \rightarrow a} \frac{d^{q-1}}{dz^{q-1}} [(z-a)^q f(z)]$$

Povezava med ravninama z in s

$$z = e^{sT}$$

Diskretna konvolucija

$$y(k) = \sum_{m=0}^k u(m) h(k-m) = \sum_{m=0}^k h(m) u(k-m) \xrightarrow{Z\text{-trans.}} Y(z) = H(z) U(z)$$

Diskretna Fourierova transformacija

$$X(mF) = \sum_{k=0}^{N-1} x(kT) e^{-\frac{j2\pi mk}{N}}$$

$$x(kT) = \frac{1}{N} \sum_{m=0}^{N-1} X(mF) e^{\frac{j2\pi mk}{N}}; \quad T = \frac{1}{f_s}, \quad F = \frac{1}{t_p}$$

Prostor stanj

$$\left. \begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k) \\ y(k) &= \mathbf{c}^T \mathbf{x}(k) + du(k) \end{aligned} \right\} \leftrightarrow \frac{Y(z)}{U(z)} = \mathbf{c}^T (z\mathbf{I} - \mathbf{A})^{-1} \mathbf{b} + d$$

Diagonalna kanonična oblika

$$\mathbf{\Lambda} = \mathbf{\Theta}^{-1} \mathbf{A} \mathbf{\Theta}$$

Odziv diskretnega sistema

$$\mathbf{x}(k) = \mathbf{A}^k \mathbf{x}(0) + \sum_{i=1}^k \mathbf{A}^{(k-i)} \mathbf{b} u(i-1); \quad \mathbf{A}^k = \mathcal{Z}^{-1} [(z\mathbf{I} - \mathbf{A})^{-1} z] = \mathbf{\Theta} \mathbf{\Lambda}^k \mathbf{\Theta}^{-1}$$

Frekvenčni odziv diskretnih sistemov

$$H(z)|_{z=e^{j\omega T}} = H(e^{j\omega T}) = A(\omega) e^{j\beta(\omega)}$$

Diskretni ekvivalenti zveznih sistemov:

- Metoda prvih diferenc:  $s \sim \frac{z-1}{T}$
- Metoda zadnjih diferenc:  $s \sim \frac{z-1}{Tz}$
- Trapezna formula:  $s \sim \frac{2}{T} \frac{z-1}{z+1}$
- Načrtovanje diskretnih filtrov:  $s \sim C \frac{1-z^{-1}}{1+z^{-1}}$ , a)  $C = \frac{2}{T}$ , b)  $C = \omega_r \text{ctg} \frac{\omega_r T}{2}$
- Metoda stopničaste invariance (prenosne funkcije):  $H(z) = (1-z^{-1}) \mathcal{Z} \left\{ \mathcal{L}^{-1} \left[ \frac{G(s)}{s} \right] \right\}_{t=kT}$
- Metoda stopn. invariance (prostor stanj):  $\mathbf{A}_{dis} = e^{\mathbf{A}_v T} \doteq \mathbf{I} + \mathbf{A}_{zv} T$ ,  $\mathbf{b}_{dis} = \int_0^T e^{\mathbf{A}_v(T-\tau)} \mathbf{b}_{zv} d\tau = \mathbf{A}_{zv}^{-1} (\mathbf{A}_{dis} - \mathbf{I}) \mathbf{b}_{zv} \doteq \mathbf{b}_{zv} T$

Vodljivost, spoznavnost

$$\mathbf{Q}_v = \begin{bmatrix} \mathbf{b} & \mathbf{A}\mathbf{b} & \dots & \mathbf{A}^{n-1}\mathbf{b} \end{bmatrix}$$

$$\mathbf{Q}_s = \begin{bmatrix} \mathbf{c}^T \\ \mathbf{c}^T \mathbf{A} \\ \vdots \\ \mathbf{c}^T \mathbf{A}^{n-1} \end{bmatrix}$$

Modificiran Routhov kriterij

$$w = \frac{z-1}{z+1}$$

Tabela Laplaceove in z-transformacije

| $x_z(t)$                  | $x(k) = x_z(kT)$            | $\mathcal{L}\{x_z(t)\}$               | $\mathcal{Z}\{x(k)\}$                                                         |
|---------------------------|-----------------------------|---------------------------------------|-------------------------------------------------------------------------------|
|                           | $\delta(kT)$                |                                       | 1                                                                             |
| 1                         | 1                           | $\frac{1}{s}$                         | $\frac{z}{z-1}$                                                               |
| $t$                       | $kT$                        | $\frac{1}{s^2}$                       | $\frac{Tz}{(z-1)^2}$                                                          |
| $t^2$                     | $k^2T^2$                    | $\frac{2}{s^3}$                       | $\frac{T^2z(z+1)}{(z-1)^3}$                                                   |
| $t^3$                     | $k^3T^3$                    | $\frac{6}{s^4}$                       | $\frac{T^3z(z^2+4z+1)}{(z-1)^4}$                                              |
| $t^n$                     | $k^nT^n$                    | $\frac{n!}{s^{n+1}}$                  | $\lim_{a \rightarrow 0} \frac{\partial^n}{\partial a^n} \frac{z}{z-e^{aT}}$   |
| $e^{-at}$                 | $e^{-akT}$                  | $\frac{1}{s+a}$                       | $\frac{z}{z-e^{-aT}}$                                                         |
| $te^{-at}$                | $kTe^{-akT}$                | $\frac{1}{(s+a)^2}$                   | $\frac{Tze^{-aT}}{(z-e^{-aT})^2}$                                             |
| $t^2e^{-at}$              | $k^2T^2e^{-akT}$            | $\frac{2}{(s+a)^3}$                   | $\frac{T^2ze^{-aT}(z+e^{-aT})}{(z-e^{-aT})^3}$                                |
| $t^ne^{at}$               | $k^nT^ne^{akT}$             | $\frac{n!}{(s-a)^{n+1}}$              | $\frac{\partial^n}{\partial a^n} \frac{z}{z-e^{aT}}$                          |
| $1 - e^{-at}$             | $1 - e^{-akT}$              | $\frac{a}{s(s+a)}$                    | $\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$                                       |
| $at - 1 + e^{-at}$        | $akT - 1 + e^{-akT}$        | $\frac{a^2}{s^2(s+a)}$                | $\frac{(aT-1+e^{-aT})z^2+(1-aTe^{-aT}-e^{-aT})z}{(z-1)^2(z-e^{-aT})}$         |
| $e^{-at} - e^{-bt}$       | $e^{-akT} - e^{-bkT}$       | $\frac{b-a}{(s+a)(s+b)}$              | $\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$                           |
| $\sin \omega_0 t$         | $\sin \omega_0 kT$          | $\frac{\omega_0}{s^2+\omega_0^2}$     | $\frac{z \sin \omega_0 T}{z^2-2z \cos \omega_0 T+1}$                          |
| $\cos \omega_0 t$         | $\cos \omega_0 kT$          | $\frac{s}{s^2+\omega_0^2}$            | $\frac{z(z-\cos \omega_0 T)}{z^2-2z \cos \omega_0 T+1}$                       |
| $e^{-at} \sin \omega_0 t$ | $e^{-akT} \sin \omega_0 kT$ | $\frac{\omega_0}{(s+a)^2+\omega_0^2}$ | $\frac{ze^{-aT} \sin \omega_0 T}{z^2-2ze^{-aT} \cos \omega_0 T+e^{-2aT}}$     |
| $e^{-at} \cos \omega_0 t$ | $e^{-akT} \cos \omega_0 kT$ | $\frac{s+a}{(s+a)^2+\omega_0^2}$      | $\frac{z^2-ze^{-aT} \cos \omega_0 T}{z^2-2ze^{-aT} \cos \omega_0 T+e^{-2aT}}$ |